SOL HW 6.3

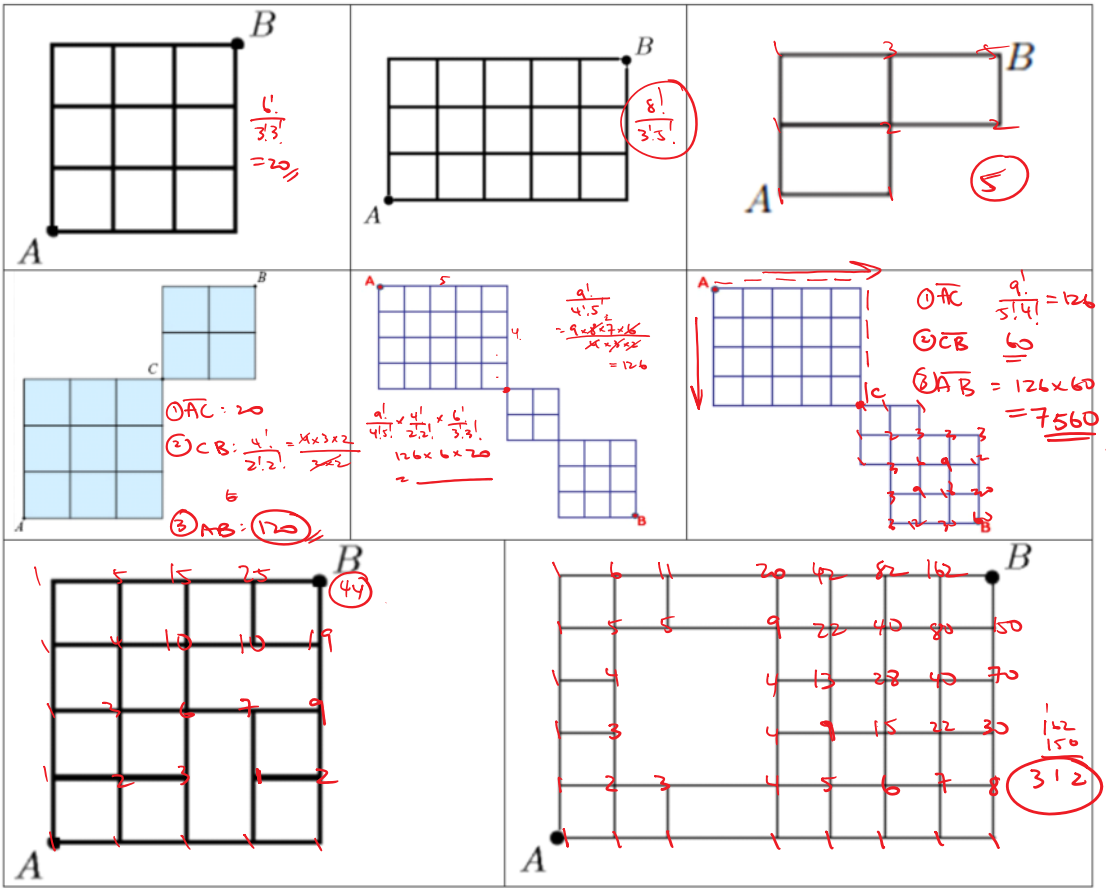
January 24, 2018 2:11 PM

Name: Key **Math 10 Enriched HW 6.3 Combinations and Pascals Triangle**

1. Simplify each of the following:

a) $\frac{8!}{3!4!5!}$	b) $\frac{13!}{5!4!3!}$	c) $\frac{8!-7!}{6!-5!}$
d) $\frac{(3!)!}{3!+4!}$	e) $\frac{(2n+3)!}{(2n-1)!}$	f) $\frac{n!-6(n-2)!}{(n-3)(n-2)!}$

2. For each of the following diagrams below, find the number of paths from Point A to B if you can only travel "up" and "right".



3. A teacher has 4 bananas, 3 apples, 5 oranges, and 7 watermelons. In how many ways can the teacher distribute these fruits to 19 students with one each?

$$\frac{19!}{4!\,3!\,5!\,7!}$$

4. The Lakers played the Golden State Warriors in a 7 game series. How many ways can the games be won by the two teams. The series is over if any team wins four games.

① Count the ways the Golden State Warriors will win, then times 2

in 4 games WWWW ①

5 games $\frac{4!}{3!}$ W ④

6 games $\frac{5!}{2!3!}$ W ⑩

7 games $\frac{6!}{3!3!}$ W ⑳

35 ways for Golden State to win.

# of ways to win altogether:

$35 \times 2 = 70$ ways

2nd option:

- There are up to 7 games, Golden State needs to win 7 of them.

W _ W _ W W _ — play only 6 games. ×2

W W W W _ _ _ — play only 4 games.

∴ $7C_4 \times 2$

$= 35 \times 2 = 70$

5. The Canucks played the Flames in a 5 game playoff series. If the Canucks won the series, how many ways can the games in the series be won?

① To win a 5 game series, a team needs to win 3 games.

1st method:

in 3 games C C C → ①

in 4 games $\frac{3!}{2!}=3$ C ③

in 5 games $\frac{4!}{2!2!}=6$ C ⑥

10 ways

2nd method

- up to 5 games are played.

C _ C _ C

$5C_3 = 10$ ways

6. How many 8 digit numbers have 3 ones, 2 fives, and 3 sixes?

1 1 5 5 6 6 6

$$\frac{8!}{3!\,2!\,3!}$$

7. If $a \neq b$ and $9Ca = 9Cb$, then what is the value of $a+b$?

$2 \neq$

$a+b=9$

$9C_2 = 9C_7$

8. If the sum of all the numbers in the nth row of Pascals triangle is 8192, then what is ${}_nC_7$ equal to?

9. How many ways can the letters in the word MINICOOPER be arranged?

10. For the AMC 12 contest, a student can advance to the AIME if they answer 17 questions correctly and leave the rest. Out of 25 questions, how many ways can a student answer 17 correctly and make the AIME?

$25C_{17}$

$25C_8$

11. In how many ways can the letters of the word STATE be scrambled in the two T's cannot be consecutive?

$\frac{5!}{2!} - 4! = 60 - 24 = 36$

All ways to arrange STATE

$4 \times 3 \times 2 \times 1$

12. The letters in APPLEPIE are rearranged and it must begin with the letter "P". How many different arrangements can there be?

13. A teacher has 5 apples, 4 bananas, and 3 watermelons. How many ways can these 12 fruits be distributed to 11 people if each person must have at least one?

AA	3A 4B 3W →
AB	4A 3B 3W →
AW	4A 4B 2W
BB	5A 2B 3W
BW	5A 3B 2W
WW	5A 4B 1W

$\frac{10!}{3!4!3!} \times 11 = \frac{11!}{3!4!3!}$

$\frac{10!}{4!3!3!} \times 11 = \frac{11!}{4!3!3!}$

$\frac{11!}{3!4!3!} + \frac{11!}{4!3!3!} + \frac{11!}{4!4!2!} + \frac{11!}{5!2!3!} + \frac{11!}{5!3!2!} + \frac{11!}{5!4!}$

$= 196,350$

14. If there are 10 bananas, 5 apples, and 4 watermelons, how many ways can the fruit be distributed to four people if each person can have only one fruit?

$3 \times 3 \times 3 \times \square$

① → ② = 2

26 → 3 = 78

Yo

① No Apples

$2 \times 2 \times 2 \times 2 = 2^4 = 16$

② 1 Apple

$4 \times 2^3 = 32$

③ 2 Apples

$4C_2 \times 4 \times 6 \times 4 = 24$

④ 3 Apples

$4 \times 2 = 8$

15. Challenge: How many ways can 10,000 bananas, 5000 apples, and 8,000 watermelon be distributed to 5,000 people if each person can have only one fruit?

$3 \; 3 \; 3 \; \ldots \; 3 = 3^{5000}$

16. Challenge: How many 15 letter arrangements of 5 A's, 5 B's, and 5 C's have no A's in the first 5 letters no B's in the next 5 letters, and no C's in the last 5 letters? AMC 12

① Look at all the ways to distribute the A's.

0B	5A's	0A's 5B's	①
1B	4A's	1A 4B's	$5C_4 \times 5C_4 \times 5C_4 = 5^3 = 125$
2B	3A's	2A's 3B's	$5C_2 \times 5C_2 \times 5C_2 = 10^3 = 1000$
3B	2A's	3A's 2B's	$5C_2 \times 5C_2 \times 5C_2 = 10^3 = 1000$
4B	1A	4A's 1B	$5C_4 \times 5C_4 \times 5C_4 = 5^3 = 125$
5B	0A's	5A's 0B's	①

2252

20. How many 15-letter arrangements of 5 A's, 5 B's, and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters, and no C's in the last 5 letters?

(A) $\sum_{k=0}^{5} \binom{5}{k}^3$ **(B)** $3^5 \cdot 2^5$ **(C)** 2^{15} **(D)** $\frac{15!}{(5!)^3}$ **(E)** 3^{15}